

Higher Code Rate and Full Diversity Space Time Block Code Based on Partial Interference Cancellation Group

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Abstract

In this paper a new systematic PIC-STBC (Partial Interference Cancellation Space Time Block Code) design is proposed for MIMO (Multi-input Multi-output) systems for any transmit antennas. The proposed code uses PIC decoding scheme to decode transmitted symbols at the receiver. The Proposed PIC group STBC design which achieves full diversity and has higher code rate compared with the best known PIC-STBC designs. As a special case for four transmit antennas our code has code rate of $5/3$ with $P=2$ while the all previously proposed PIC-STBC designs have the code rate of $4/3$. Also, the new code has complexity decoding the same as conventional PIC group STBC design. Simulation results show that the proposed code achieves full diversity and gives higher coding gain which results into lower BER (Bit Error Rate) compared with the all previously proposed PIC-STBC designs.

Keywords: MIMO system, STBC, PIC decoder, full diversity, BER.

1. Introduction

In Multi-input Multi-output (MIMO) systems, Space Time Block Code (STBC) designs are attractive approach to mitigate channel fading. This brought to MIMO systems widely used in wireless communications followed by an interesting area of STBC design. Orthogonal STBC (OSTBC) design is one of the most attractive STBC designs. OSTBC design achieves full diversity with a sample pair-wise maximum likelihood (ML) decoding [1-2]. OSTBC design has a low code rate that cannot be larger than $3/4$ symbol per channel use (pcu) for more than two transmit antennas [3]. To address the problem of low rate of OSTBC design, high rate STBC designs based on ML decoding has been proposed in [4-8]. Most of those codes obtain full diversity and full rate, i.e., $L=M$ symbol pcu for M transmit antennas. Unfortunately, computation complexity of ML decoding exponentially increases with the number of symbols encoded in the code matrix.

1.1 Related Works

To address the problem of higher complexity of ML decoding, STBC designs with linear receivers such as zero forcing (ZF) and minimum mean square error (MMSE) have been presented [9-12]. Those codes have symbol-by-symbol decoding complexity and give full diversity. However, their code rate is lower than one symbol pcu. In order to address the decoding complexity and code rate tradeoff, PIC group decoding was proposed and its design criterion was also derived [13]. Later, a systematic STBC design achieving full diversity with partial interference cancellation (PIC) group decoding was proposed in [14]. However, decoding complexity of STBC design in [14] is equivalent with ML decoding, complexity of M symbols. Furthermore, a systematic design which gives full diversity with PIC and PIC- successive interference cancellation (PIC- SIC) decoding was proposed in [15].

To further reduce decoding complexity, two STBC designs with PIC decoder has been designed [16-17]. The code in [16] achieves full diversity and reduces decoding complexity, i.e., half of that in [13]. This code can be viewed as an Alamouti codeword matrix, i.e., each entry of the conventional Alamouti codeword matrix is replaced by a Toeplitz matrix. Likewise STBC design in [16], the design in [17] is coordinate interleaved orthogonal design (CIOD) [18] and its entry replaced by an elementary matrix, which is then designed as a block of multiple diagonal layers of algebraic STBC design. It should be mentioned that decoding complexity in [17] is half of that in [16]. In [17] it has been shown that all above described PIC-STBC designs represents the same bit error rate (BER) performance for different constellation sizes. Recently, PIC-STBC designs widely has been applied in MIMO multi-user systems in order to reduce symbol decoding complexity [19-20].

1.2 Our Contribution

In this paper, we propose a new systematic PIC decoder-based STBC design for MIMO system for any transmit antennas. Our goal is to increase code rate and improve BER performance. Recently, Γ -group-decodable STBC design has been proposed in [21]. Our proposed code can be viewed as an unbalanced 2-group-decodable STBC design. In fact, each entry of unbalanced 2-group-decodable STBC design is replaced by real-valued diagonal multilayer algebraic STBC design. The new designed code has decoding complexity $O(A^M)$ where M is number of the transmit antennas and $A=|S|$, S is the complex constellation, which is the same as the code in [13] and higher than [16] and [17]. It should be mentioned that the real-valued linear transform matrices are used for real and imaginary part of the transmitted symbols. The code rate of our proposed code is $R = \frac{5}{4} \times \frac{MP}{M+2P-2}$ higher than the previously full diversity PIC-STBC designs in [13] and [16-17]. For instance, for $M=4$ with $P=2$ the proposed code has the code rate of $5/3$ while the PIC-STBCs in [13] and [16-17] have the code rate of $4/3$. Simulation results prove that the proposed PIC-STBC design achieves full diversity and compared with the codes in [13] and [16-17] gives higher coding gain and better BER performance.

The rest of the paper is organized as follows. In Section II the system model is explained, also, Γ -group decodable STBC Conditions, unbalanced Γ -group decodable STBC, and PIC group STBC are described. Section III comprises three subsections: 1) Systematic Proposed Code Design, 2) Code design examples, and 3) code properties. In Section IV, the performance of the introduced code is evaluated and compared with other codes. Finally, Section V concludes the paper.

Notations: Small letters, bold letters, and bold capital letters will designate scalars, vectors, and matrices, respectively. $\mathbf{X}^H$, $\mathbf{X}^t$, $\mathbf{X}^*$, and $\text{tr}(\mathbf{X})$ denote the conjugate-transpose, transpose, complex conjugate, and trace of the matrix $\mathbf{X}$, respectively.

2. System Model

2.1 Signal Model

Consider an MIMO system with M transmit antennas and N receiver antennas with quasi-static flat fading of block length T . It is assumed channel state information (CSI) is known at receiver and unknown at transmitter. The transmit-receive signal relationship be presented as [21]:

$$\mathbf{X}(\mathbf{s})_{T \times M} = \sum_{l=1}^L \mathbf{C}_l s_l \quad (1)$$

Where $s_l \in \mathbb{R}$ are real valued symbol representing the real and imaginary components of complex constellation symbols, $\mathbf{C}_l \in \mathbb{C}^{T \times M}$ are called dispersion matrices. The transmit-receive signal relationship can be written as [21]:

$$(\tilde{\mathbf{r}}_1 \tilde{\mathbf{r}}_2 \cdots \tilde{\mathbf{r}}_N) = \sqrt{q} \mathbf{X}(\mathbf{s}) \tilde{\mathbf{H}} + \tilde{\mathbf{W}} \quad (2)$$

where the receive signals $\tilde{\mathbf{r}}_{tn}$ of the n th receive antenna at time t can be arranged in a $T \times M$ matrix $(\tilde{\mathbf{r}}_1 \tilde{\mathbf{r}}_2 \cdots \tilde{\mathbf{r}}_N) = (\tilde{\mathbf{r}}_{tn})$. The normalized q ($q = \rho/M$) is to ensure that the SNR (at the receiver) is independent of the number of the transmit antennas. $\mathbf{X}(\mathbf{s})_{T \times M}$ is the encoded matrix of transmitted symbols that are drawn complex constellation. $\tilde{\mathbf{H}}_{M \times N} = (\tilde{\mathbf{h}}_1 \tilde{\mathbf{h}}_2 \cdots \tilde{\mathbf{h}}_N)$ contains all the channel coefficients with zero mean and unit variance $\tilde{h}_{mn}$. $\tilde{\mathbf{Z}}_{T \times N} = (\tilde{\mathbf{z}}_1 \tilde{\mathbf{z}}_2 \cdots \tilde{\mathbf{z}}_N) = (\tilde{\mathbf{z}}_{tn})$ is the noise matrix. The entries $\tilde{z}_{tn}$ are assumed to be independently, identically distributed (i.i.d) complex Gaussian random variables with the probability density function (pdf) $CN(0, 1)$. After some manipulation, equivalent signal model is obtained from (2) as [21],

$$\mathbf{r} = \sqrt{q} \mathbf{H} \mathbf{s} + \mathbf{w}, \quad (3)$$

where $\mathbf{r}_{TN \times 1}$ is a received signal vector, $\mathbf{w}_{TN \times 1}$ is a complex noise vector, and $\mathbf{H}_{TN \times L}$ is an equivalent channel matrix.

2.2 Γ -group decodable STBC conditions [21]

In this subsection we introduce the necessary conditions of Γ -group decodable STBC designs. Assume that the transmitted symbols can be separated into Γ -groups and each group has real L_i symbols, then $\sum_{i=1}^{\Gamma} L_i = L$. Let the set of indexes of symbols in the i th group be denoted as Θ_i . For an STBC to be Γ -group decodable, two conditions should be satisfied [21]:

- 1) $\mathbf{h}_p^\dagger \mathbf{h}_q = 0$ where $p \in \Theta_{i_1}$, $q \in \Theta_{i_2}$ and $i_1 \neq i_2$;
- 2) $\text{rank}(\mathbf{H}_i) = L_i$ where $\mathbf{H}_i = (\mathbf{h}_{i_1} \mathbf{h}_{i_2} \cdots \mathbf{h}_{i_{L_i}})$,
 $i_k \in \Theta_i$, $k = 1, 2, \dots, L_i$, $i = 1, 2, \dots, \Gamma$.

Condition 1) means that the STBC is group-decodable and condition 2) guarantees that no decoder of any group is rank deficient [21].

Unbalanced Γ -group decodable STBC: The Γ -group decodable STBC has been presented. Now we represent unbalanced Γ -group decodable STBC. Consider the unbalanced 2-group decodable STBC with $L_1=1$ and $L_2=L-1$. Then, we have [21],

$$\mathbf{C}_1^H \mathbf{C}_l = -\mathbf{C}_l^H \mathbf{C}_1 \quad \text{for } l=2,3,\dots,L. \quad (4)$$

As an example, here we introduce an unbalanced 2-group decodable STBC's dispersion matrices [21]:

$$\left\{ \begin{aligned} \mathbf{C}_1 &= \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}; \\ \mathbf{C}_2 &= \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \mathbf{C}_3 = \begin{pmatrix} j & -j \\ j & j \end{pmatrix}, \mathbf{C}_4 = \begin{pmatrix} j & j \\ -j & j \end{pmatrix}, \mathbf{C}_5 = \begin{pmatrix} j & j \\ j & -j \end{pmatrix} \end{aligned} \right\} \quad (5)$$

Hence, an unbalanced 2-group-decodable STBC with the dispersion matrices in (5) for 2 transmit antennas can be obtained as [21]:

$$\mathbf{X}_{2 \times 2}(\mathbf{s}) = \begin{pmatrix} s_1 - s_2 + js_3 + js_4 + js_5 & s_1 + s_2 - js_3 + js_4 + js_5 \\ s_1 + s_2 + js_3 - js_4 + js_5 & -s_1 + s_2 + js_3 + js_4 - js_5 \end{pmatrix} \quad (6)$$

where s_1 is in the first group, while s_2 to s_5 are in the second group.

2.3 PIC Group Decoding [13]

This subsection describes the PIC group decoding procedure. As mentioned, in [13], new PIC group decoding scheme was proposed. To address the code rate and decoding complexity tradeoff while achieving full diversity, PIC group decoding scheme is introduced. The PIC group decoding scheme is presented in below.

In the PIC group decoding the equivalent channel matrix $\mathbf{H}_{TN \times L}$ in (3) is divided into P column group $\{\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_P\}$ with l_p column for group $\mathbf{G}_p$, $p=1,2,\dots,P$ and $\sum_{p=1}^P l_p = 2L$. Then, for group $\mathbf{G}_p$ a group ZF is applied to repress interference from all the other groups, i.e., $\{\mathbf{G}_1, \dots, \mathbf{G}_{p-1}, \mathbf{G}_{p+1}, \dots, \mathbf{G}_P\}$. afterwards, ML decoding is used to jointly decode symbols $\mathbf{s}_p$ corresponding to the group $\mathbf{G}_p$. With these specifications, (2) can be written as [16]

$$\mathbf{r}' = \sum_{p=1}^P \mathbf{G}_p \mathbf{s}_p + \mathbf{w} \quad (7)$$

Suppose we want to decode the symbols embedded in the group $\mathbf{s}_p$ using PIC group decoding. The PIC group decoding has two steps.

Step 1: linear interference cancellation with a suitable choice of matrix $\mathbf{Q}_p$ is used in order to completely repress the interference from all other groups [1], i.e., $\mathbf{Q}_p \mathbf{G}_q = \mathbf{0}$, $\forall q \neq p$ and $q=1,2,\dots,P$. Then we have

$$\mathbf{z}_p \equiv \mathbf{Q}_p \mathbf{r}' = \sqrt{q} \mathbf{Q}_p \mathbf{G}_p \mathbf{s}_p + \mathbf{Q}_p \mathbf{w}, \quad p=1,2,\dots,P \quad (8)$$

where the interference cancellation matrix $\mathbf{Q}_p$ can be chosen as follows [1],

$$\mathbf{Q}_p = \mathbf{I} - \mathbf{G}_p^c ((\mathbf{G}_p^c)^H \mathbf{G}_p^c)^{-1} (\mathbf{G}_p^c)^H, \quad p=1,2,\dots,P \quad (9)$$

with $\mathbf{G}_p^c = (\mathbf{G}_1, \dots, \mathbf{G}_{p-1}, \mathbf{G}_{p+1}, \dots, \mathbf{G}_P)$.

Step 2: the symbols in the group $\mathbf{s}_p$ are decoded with the ML decoding algorithm as follows,

$$\hat{\mathbf{s}}_p = \arg \min_{\mathbf{s}_p \in A^{l_p}} \|\mathbf{z}_p - \sqrt{q} \mathbf{Q}_p \mathbf{G}_p \mathbf{s}_p\|. \quad (10)$$

Note that the interference cancellation in step 1 mainly involves with linear matrix computations, whose computational complexity is small compared with the joint decoding with an exhaustive search of all candidate symbols in step 2. To evaluate the decoding complexity of the PIC group decoding, we mainly focus on the computational complexity of the joint decoding of each group under the PIC group decoding scheme, i.e., complexity of step 2. The joint decoding complexity can be characterized by the number of Ferbenius norms calculated in the decoding process. In the mentioned decoding algorithm the complexity is $O = \sum_{p=1}^P A^{l_p}$. It can be seen that the PIC group decoding provides a flexible decoding complexity which can vary from the ZF decoding complexity LA to the ML decoding complexity A^L [17].

Difference between the Γ -group decodable STBC design and PIC group STBC design: we should mention that in Γ -group decodable STBC design the transmitted symbols are divided into groups, the symbols of each group are orthogonal to symbols of other groups. Therefore, the minimization of ML criterion is equivalent to minimize whole symbols in each group, jointly, i.e.,

$$\arg \min_s \|\mathbf{r} - \sqrt{q} \mathbf{H} \mathbf{s}\| = \sum_{i=1}^F \left\| \mathbf{r} - \sqrt{q} \mathbf{H}_i (s_1 \ s_2 \ \dots \ s_{l_i})^T \right\| \quad (11)$$

However, in PIC group decodable STBC design the transmitted symbols in each group interfere with the symbols of other groups. Therefore, before applying the ML criterion interference from other groups cancelled as shown in (8) and (9). Then, the symbols in each group decoded as expressed in (10).

3. The Proposed PIC Group STBC Design

In this section, new systematic STBC design based on the PIC group decoding will be represented. Then, two code design examples are given. At the end, properties of the proposed code have been described.

3.1 Systematic Proposed Code Design

Suppose $T = 2(\frac{M}{2} + P - 1)$ and M is even for any given M .

Our proposed STBC, $\Psi_{M,T,P}$, is:

$$\Psi_{M,T,P} = \sum_{\gamma=I}^{\Pi} \sum_{k=1}^{L_{\gamma}} \mathbf{C}_{\gamma,k} \otimes \mathbf{X}_{\gamma,k}^{\frac{T \times M}{2}}(\mathbf{s}) \text{ for } \gamma = I, \Pi, k=1,2,\dots,L_{\gamma} \quad (12)$$

where $\mathbf{C}_{\gamma,k} \in \mathbb{C}^{3 \times 2}$ is called dispersion matrix and has been represented in (5). $\mathbf{X}_{\gamma,k}^{\frac{T \times M}{2}}(\mathbf{s}) \in \mathbb{R}^{\frac{T \times M}{2}}$ is equal [13]:

(13)

with p th diagonal layer from left to right is a vector with

$$\frac{M}{2} \times 1 \text{ size and defined as } \mathbf{x}_{\gamma,k}^p = \begin{pmatrix} x_{\gamma,k}^{p,1} & x_{\gamma,k}^{p,2} & \dots & x_{\gamma,k}^{p,\frac{M}{2}} \end{pmatrix}^t$$

for $p = 1, 2, \dots, P$ where vector $\mathbf{x}_{\gamma,k}^p$ is obtained as:

$$\mathbf{x}_{\gamma,k}^p = \mathbf{\Theta}_{\frac{M}{2} \times \frac{M}{2}} \mathbf{s}_{\gamma,k}^p \quad (14)$$

with $\mathbf{\Theta} \in \mathbb{R}^{\frac{M}{2} \times \frac{M}{2}}$ is linear transform matrix [22], and vector

$\mathbf{s}_{\gamma,k}^p \in \mathbb{R}^{\frac{M}{2} \times 1}$ is either real or imaginary part of the symbols drawn from constellation. Then, we consider rate > 1 , unbalanced 2-group decodable STBC design was shown in (6). Finally, by considering equation (12) the systematic proposed PIC group STBC design is given as,

$$\Psi_{M,T,P} = \sum_{\gamma=I}^{\Pi} \sum_{k=1}^{L_{\gamma}} \mathbf{C}_{\gamma,k} \otimes \mathbf{X}_{\gamma,k}^{\frac{T \times M}{2}}(\mathbf{s}) = \begin{pmatrix} \mathbf{X}_{I,1} - \mathbf{X}_{\Pi,1} + j\mathbf{X}_{\Pi,2} + j\mathbf{X}_{\Pi,3} + j\mathbf{X}_{\Pi,4} & \mathbf{X}_{I,1} + \mathbf{X}_{\Pi,1} - j\mathbf{X}_{\Pi,2} + j\mathbf{X}_{\Pi,3} + j\mathbf{X}_{\Pi,4} \\ \mathbf{X}_{I,1} + \mathbf{X}_{\Pi,1} + j\mathbf{X}_{\Pi,2} - j\mathbf{X}_{\Pi,3} + j\mathbf{X}_{\Pi,4} & -\mathbf{X}_{I,1} + \mathbf{X}_{\Pi,1} + j\mathbf{X}_{\Pi,2} + j\mathbf{X}_{\Pi,3} - j\mathbf{X}_{\Pi,4} \end{pmatrix} \quad (15)$$

where $\mathbf{X}_{\gamma,k}^{\frac{T \times M}{2}}$ is defined in (13). In the next subsection we give two examples of the proposed code.

3.2 Code Design Examples

In this subsection we provide two proposed STBC examples for four and six transmit antennas, respectively.

3.2.1 Four Transmit Antennas, $M = 4, T = 6$

Suppose $P = 2$ [13]. According to (12) we obtain,

$$\Psi_{4,6,2} = \sum_{\gamma=I}^{\Pi} \sum_{k=1}^{L_{\gamma}} \mathbf{C}_{\gamma,k} \otimes \mathbf{X}_{\gamma,k}^{3 \times 2}(\mathbf{s}) \quad (16)$$

where $\mathbf{C}_{\gamma,k}$ is represented in (5) and STBC matrix $\mathbf{X}_{\gamma,k}^{3 \times 2}(\mathbf{s})$ is [13]

$$\mathbf{X}_{\gamma,k}^{3 \times 2}(\mathbf{s}) = \begin{pmatrix} x_{\gamma,k}^{1,1} & 0 \\ x_{\gamma,k}^{2,1} & x_{\gamma,k}^{1,2} \\ 0 & x_{\gamma,k}^{2,2} \end{pmatrix} \text{ for } \gamma = I, \Pi \text{ and } L_I = 1, L_{\Pi} = 4 \quad (17)$$

with regarding to (14) diagonal layers of $\mathbf{X}_{\gamma,k}^{3 \times 2}(\mathbf{s})$ for any γ and k is calculated as below.

$$\left\{ \mathbf{x}_{I,1}^1 = \begin{pmatrix} x_{I,1}^{1,1} \\ x_{I,1}^{1,2} \end{pmatrix} = \mathbf{\Theta}_{2 \times 2} \begin{pmatrix} s_1^R \\ s_2^R \end{pmatrix}, \quad \mathbf{x}_{I,1}^2 = \begin{pmatrix} x_{I,1}^{2,1} \\ x_{I,1}^{2,2} \end{pmatrix} = \mathbf{\Theta}_{2 \times 2} \begin{pmatrix} s_1^I \\ s_2^I \end{pmatrix} \right\}; \quad (18)$$

where

$$\mathbf{\Theta}_{2 \times 2} = \begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix} \quad (19)$$

with $\theta = 1.02$ [13].

Therefore, the proposed STBC design $\Psi_{4,6,2}$ is

$$\Psi_{4,6,2} = \begin{pmatrix} x_{I,1}^{1,1} - x_{\Pi,1}^{1,1} + jx_{\Pi,2}^{1,1} + jx_{\Pi,3}^{1,1} + jx_{\Pi,4}^{1,1} & 0 & x_{I,1}^{1,2} - x_{\Pi,1}^{1,2} + jx_{\Pi,2}^{1,2} + jx_{\Pi,3}^{1,2} + jx_{\Pi,4}^{1,2} & 0 \\ x_{I,1}^{2,1} - x_{\Pi,1}^{2,1} + jx_{\Pi,2}^{2,1} + jx_{\Pi,3}^{2,1} + jx_{\Pi,4}^{2,1} & x_{I,1}^{2,2} - x_{\Pi,1}^{2,2} + jx_{\Pi,2}^{2,2} + jx_{\Pi,3}^{2,2} + jx_{\Pi,4}^{2,2} & x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} + jx_{\Pi,2}^{1,2} - jx_{\Pi,3}^{1,2} + jx_{\Pi,4}^{1,2} & 0 \\ 0 & x_{I,1}^{2,2} - x_{\Pi,1}^{2,2} + jx_{\Pi,2}^{2,2} + jx_{\Pi,3}^{2,2} + jx_{\Pi,4}^{2,2} & x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} + jx_{\Pi,2}^{1,2} - jx_{\Pi,3}^{1,2} + jx_{\Pi,4}^{1,2} & 0 \\ x_{I,1}^{1,1} + x_{\Pi,1}^{1,1} + jx_{\Pi,2}^{1,1} - jx_{\Pi,3}^{1,1} + jx_{\Pi,4}^{1,1} & 0 & x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} - jx_{\Pi,2}^{1,2} + jx_{\Pi,3}^{1,2} + jx_{\Pi,4}^{1,2} & 0 \\ x_{I,1}^{2,1} + x_{\Pi,1}^{2,1} - jx_{\Pi,2}^{2,1} + jx_{\Pi,3}^{2,1} + jx_{\Pi,4}^{2,1} & x_{I,1}^{2,2} + x_{\Pi,1}^{2,2} - jx_{\Pi,2}^{2,2} + jx_{\Pi,3}^{2,2} + jx_{\Pi,4}^{2,2} & x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} - jx_{\Pi,2}^{1,2} + jx_{\Pi,3}^{1,2} + jx_{\Pi,4}^{1,2} & 0 \\ 0 & x_{I,1}^{2,2} + x_{\Pi,1}^{2,2} - jx_{\Pi,2}^{2,2} + jx_{\Pi,3}^{2,2} + jx_{\Pi,4}^{2,2} & x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} - jx_{\Pi,2}^{1,2} + jx_{\Pi,3}^{1,2} + jx_{\Pi,4}^{1,2} & 0 \\ -x_{I,1}^{1,1} + x_{\Pi,1}^{1,1} + jx_{\Pi,2}^{1,1} + jx_{\Pi,3}^{1,1} - jx_{\Pi,4}^{1,1} & 0 & -x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} + jx_{\Pi,2}^{1,2} + jx_{\Pi,3}^{1,2} - jx_{\Pi,4}^{1,2} & 0 \\ -x_{I,1}^{2,1} + x_{\Pi,1}^{2,1} + jx_{\Pi,2}^{2,1} + jx_{\Pi,3}^{2,1} - jx_{\Pi,4}^{2,1} & 0 & -x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} + jx_{\Pi,2}^{1,2} + jx_{\Pi,3}^{1,2} - jx_{\Pi,4}^{1,2} & 0 \\ 0 & 0 & -x_{I,1}^{1,2} + x_{\Pi,1}^{1,2} + jx_{\Pi,2}^{1,2} + jx_{\Pi,3}^{1,2} - jx_{\Pi,4}^{1,2} & 0 \end{pmatrix} \quad (20)$$

The code rate of $\Psi_{4,6,2}$ is 5/3 higher than the code rate of the codes in [13] and [16-17] for four transmit antennas with the code rate of 4/3.

3.2.2 Six Transmit Antennas, $M = 6, T = 8$

Suppose $P = 2$. Again, according to (12) we have

$$\Psi_{6,8,2} = \sum_{\gamma=1}^H \sum_{k=1}^{L_{\gamma}} \mathbf{C}_{\gamma,k} \otimes \mathbf{X}_{\gamma,k}^{4 \times 3}(\mathbf{s}) \quad (21)$$

where the diagonal layers of $\mathbf{X}_{\gamma,k}^{4 \times 3}(\mathbf{s})$ is obtained like $\mathbf{X}_{\gamma,k}^{3 \times 2}(\mathbf{s})$ in (18). The real linear transform matrix for $M = 6, \theta_{3 \times 3}$, is [22]

$$\theta_{3 \times 3} = \begin{pmatrix} 0.745 & -0.582 & -0.326 \\ -0.326 & 0.745 & -0.582 \\ 0.582 & 0.326 & 0.745 \end{pmatrix} \quad (22)$$

Thus, the code $\Psi_{6,8,2}$ obtains as,

$$\Psi_{6,8,2} = \begin{pmatrix} x_{1,1}^{1,1} - x_{11,1}^{1,1} + jx_{11,2}^{1,1} + jx_{11,3}^{1,1} + jx_{11,4}^{1,1} & 0 & x_{1,1}^{1,2} - x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & 0 & x_{1,1}^{1,3} - x_{11,1}^{1,3} + jx_{11,2}^{1,3} + jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & -x_{1,1}^{1,4} + x_{11,1}^{1,4} + jx_{11,2}^{1,4} + jx_{11,3}^{1,4} - jx_{11,4}^{1,4} & 0 \\ x_{1,1}^{2,1} - x_{11,1}^{2,1} + jx_{11,2}^{2,1} + jx_{11,3}^{2,1} + jx_{11,4}^{2,1} & x_{1,1}^{1,2} - x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & x_{1,1}^{2,2} - x_{11,1}^{2,2} + jx_{11,2}^{2,2} + jx_{11,3}^{2,2} + jx_{11,4}^{2,2} & 0 & x_{1,1}^{2,3} - x_{11,1}^{2,3} + jx_{11,2}^{2,3} + jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & -x_{1,1}^{2,4} + x_{11,1}^{2,4} + jx_{11,2}^{2,4} + jx_{11,3}^{2,4} - jx_{11,4}^{2,4} & 0 \\ 0 & x_{1,1}^{1,2} - x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & x_{1,1}^{2,2} - x_{11,1}^{2,2} + jx_{11,2}^{2,2} + jx_{11,3}^{2,2} + jx_{11,4}^{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_{1,1}^{1,1} + x_{11,1}^{1,1} + jx_{11,2}^{1,1} - jx_{11,3}^{1,1} + jx_{11,4}^{1,1} & 0 & x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} - jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & 0 & x_{1,1}^{1,3} + x_{11,1}^{1,3} + jx_{11,2}^{1,3} - jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & -x_{1,1}^{1,4} + x_{11,1}^{1,4} + jx_{11,2}^{1,4} + jx_{11,3}^{1,4} - jx_{11,4}^{1,4} & 0 \\ x_{1,1}^{2,1} + x_{11,1}^{2,1} + jx_{11,2}^{2,1} - jx_{11,3}^{2,1} + jx_{11,4}^{2,1} & x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} - jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} - jx_{11,3}^{2,2} + jx_{11,4}^{2,2} & 0 & x_{1,1}^{2,3} + x_{11,1}^{2,3} + jx_{11,2}^{2,3} - jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & -x_{1,1}^{2,4} + x_{11,1}^{2,4} + jx_{11,2}^{2,4} + jx_{11,3}^{2,4} - jx_{11,4}^{2,4} & 0 \\ 0 & x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} - jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} - jx_{11,3}^{2,2} + jx_{11,4}^{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{1,1}^{1,1} + x_{11,1}^{1,1} + jx_{11,2}^{1,1} - jx_{11,3}^{1,1} + jx_{11,4}^{1,1} & x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} - jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & 0 & x_{1,1}^{1,3} + x_{11,1}^{1,3} + jx_{11,2}^{1,3} - jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & -x_{1,1}^{1,4} + x_{11,1}^{1,4} + jx_{11,2}^{1,4} + jx_{11,3}^{1,4} - jx_{11,4}^{1,4} & 0 \\ 0 & x_{1,1}^{2,1} + x_{11,1}^{2,1} + jx_{11,2}^{2,1} - jx_{11,3}^{2,1} + jx_{11,4}^{2,1} & x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} - jx_{11,3}^{2,2} + jx_{11,4}^{2,2} & 0 & x_{1,1}^{2,3} + x_{11,1}^{2,3} + jx_{11,2}^{2,3} - jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & -x_{1,1}^{2,4} + x_{11,1}^{2,4} + jx_{11,2}^{2,4} + jx_{11,3}^{2,4} - jx_{11,4}^{2,4} & 0 \\ x_{1,1}^{1,3} - x_{11,1}^{1,3} + jx_{11,2}^{1,3} + jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & x_{1,1}^{1,2} - x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & 0 & x_{1,1}^{1,3} - x_{11,1}^{1,3} + jx_{11,2}^{1,3} + jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & -x_{1,1}^{1,4} + x_{11,1}^{1,4} + jx_{11,2}^{1,4} + jx_{11,3}^{1,4} - jx_{11,4}^{1,4} & 0 \\ x_{1,1}^{2,3} - x_{11,1}^{2,3} + jx_{11,2}^{2,3} + jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & x_{1,1}^{2,2} - x_{11,1}^{2,2} + jx_{11,2}^{2,2} + jx_{11,3}^{2,2} + jx_{11,4}^{2,2} & 0 & x_{1,1}^{2,3} - x_{11,1}^{2,3} + jx_{11,2}^{2,3} + jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & -x_{1,1}^{2,4} + x_{11,1}^{2,4} + jx_{11,2}^{2,4} + jx_{11,3}^{2,4} - jx_{11,4}^{2,4} & 0 \\ 0 & -x_{1,1}^{1,4} + x_{11,1}^{1,4} + jx_{11,2}^{1,4} + jx_{11,3}^{1,4} - jx_{11,4}^{1,4} & -x_{1,1}^{2,4} + x_{11,1}^{2,4} + jx_{11,2}^{2,4} + jx_{11,3}^{2,4} - jx_{11,4}^{2,4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -x_{1,1}^{1,4} + x_{11,1}^{1,4} + jx_{11,2}^{1,4} + jx_{11,3}^{1,4} - jx_{11,4}^{1,4} & -x_{1,1}^{2,4} + x_{11,1}^{2,4} + jx_{11,2}^{2,4} + jx_{11,3}^{2,4} - jx_{11,4}^{2,4} & 0 & 0 & 0 & 0 & 0 \\ x_{1,1}^{1,3} + x_{11,1}^{1,3} + jx_{11,2}^{1,3} - jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} - jx_{11,3}^{1,2} + jx_{11,4}^{1,2} & 0 & x_{1,1}^{1,3} + x_{11,1}^{1,3} + jx_{11,2}^{1,3} - jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & -x_{1,1}^{1,4} + x_{11,1}^{1,4} + jx_{11,2}^{1,4} + jx_{11,3}^{1,4} - jx_{11,4}^{1,4} & 0 \\ x_{1,1}^{2,3} + x_{11,1}^{2,3} + jx_{11,2}^{2,3} - jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} - jx_{11,3}^{2,2} + jx_{11,4}^{2,2} & 0 & x_{1,1}^{2,3} + x_{11,1}^{2,3} + jx_{11,2}^{2,3} - jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & -x_{1,1}^{2,4} + x_{11,1}^{2,4} + jx_{11,2}^{2,4} + jx_{11,3}^{2,4} - jx_{11,4}^{2,4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} - jx_{11,4}^{1,2} & 0 & -x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} - jx_{11,4}^{1,2} & 0 & -x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} - jx_{11,4}^{1,2} & 0 & -x_{1,1}^{1,2} + x_{11,1}^{1,2} + jx_{11,2}^{1,2} + jx_{11,3}^{1,2} - jx_{11,4}^{1,2} & 0 \\ -x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} + jx_{11,3}^{2,2} - jx_{11,4}^{2,2} & 0 & -x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} + jx_{11,3}^{2,2} - jx_{11,4}^{2,2} & 0 & -x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} + jx_{11,3}^{2,2} - jx_{11,4}^{2,2} & 0 & -x_{1,1}^{2,2} + x_{11,1}^{2,2} + jx_{11,2}^{2,2} + jx_{11,3}^{2,2} - jx_{11,4}^{2,2} & 0 \\ 0 & -x_{1,1}^{1,3} + x_{11,1}^{1,3} + jx_{11,2}^{1,3} - jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & -x_{1,1}^{2,3} + x_{11,1}^{2,3} + jx_{11,2}^{2,3} - jx_{11,3}^{2,3} + jx_{11,4}^{2,3} & 0 & -x_{1,1}^{1,3} + x_{11,1}^{1,3} + jx_{11,2}^{1,3} - jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 & -x_{1,1}^{1,3} + x_{11,1}^{1,3} + jx_{11,2}^{1,3} - jx_{11,3}^{1,3} + jx_{11,4}^{1,3} & 0 \end{pmatrix} \quad (23)$$

The code rate of $\Psi_{6,8,2}$ is 15/8 which is higher than the code rate of the codes in [13] and [16-17] for six transmit antennas with the code rate of 3/2.

3.3 The Proposed PIC Group STBC Design Properties

1) *Codeword matrix*: the main matrix of the proposed code has 2×2 size. Each symbol s_l for $l = 1, 2, \dots, L$ is

replaced by a diagonal algebraic multilayer STBC matrix. Note that, the proposed code is different from the codes in [16] and [17]. In [16] the main matrix is a classical Alamouti STBC matrix; moreover, in [17] the main matrix is a 2×2 CIOD STBC matrix [18]. Also, To design the new systematic PIC group STBC for M odd, we may design the code for $M+1$ transmit antennas then select first M columns of the designed code.

2) *Code rate*: The proposed STBC's code rate is

$$R = \frac{L}{T} = \frac{5}{4} \times \frac{MP}{M + 2P - 2} \quad (24)$$

which is higher than the code rate of the designed codes in [13] and [16-17].

3) *Decoding complexity*: ML decoding of the main unbalanced 2-group decodable STBC has joint two complex symbols (four real symbol) decoding complexity [21]. Therefore, the decoding complexity depends on the decoding complexity of the conventional PIC group STBC and unbalanced 2-group decodable STBC. Since the decoding complexity of the conventional PIC group STBC, $\mathbf{X}_{\gamma,k}^{T \times \frac{M}{2}}$, is $\frac{M}{2}$, thus,

$$O = A^M, \quad A = |S| \quad (25)$$

where S represents complex constellation. Therefore, the decoding complexity of the proposed PIC group STBC is M complex symbol the same as the code in [13] and higher than the codes in [16] and [17].

4. Simulation Results

In this section, the simulation results of the proposed scheme, $\Psi_{M,T,P}$, are shown for $M = 4$. It is assumed that the amplitudes of the fading from each transmit antennas to the receive antennas are mutually uncorrelated Rayleigh-distributed and the receiver has perfect knowledge of channel. We first show BER performance of the code proposed in this paper for four transmit antennas and four receive antennas and compared it with the one proposed in [13] and [16-17]. We consider the STBC proposed in (20) and compared it with Guo-Xia's code in [1, Eq. (40)], $\mathbf{B}_{4,6,2}$ in [16, Eq. (37)], and $\Phi_{4,6,2}$ [17, Eq. (12)]. Note that all codes have the same code rate of 4/3 except our proposed code in (20) that has code rate of 5/3. In order to have the same bandwidth efficiency for all codes, 32-QAM constellation is used for the transmitted symbols in Guo-Xia's code, $\mathbf{B}_{4,6,2}$, and $\Phi_{4,6,2}$ are and 16-QAM constellation is used in our proposed code. Therefore, all codes have the same bandwidth efficiency of

20/3 bits per channel use (20/3bcpu). As can be observed from fig. 1 all BER curves have the same order, thus, all the codes achieve full diversity.

Also, from fig. 1 it is clear that the proposed code in this paper outperforms the other STBC design which have the same performance. As an example at SNR 14 dB, the proposed code gives BER of 0.0029 while the other codes give 0.0073. Hence, at high SNR region the proposed STBC design in this paper performs about 1.5 dB better than the best known PIC-based STBC design. In fact, we relaxed the decoding complexity and focus on the lower BER due to higher coding gain. In this paper, we proposed a new systematic PIC group STBC design for MIMO system. The proposed code is constructed by embedding conventional PIC group STBC design in the entries of the unbalanced 2-group decodable STBC for any number of transmitted antennas. The new systematic STBC design gives higher code rate compared with the best known PIC group STBC designs. However, the decoding complexity of the transmitted symbols of our proposed code is the same as the conventional PIC group STBC design and higher than the $\mathbf{B}_{M,T,P}$, and $\Phi_{M,T,P}$. In

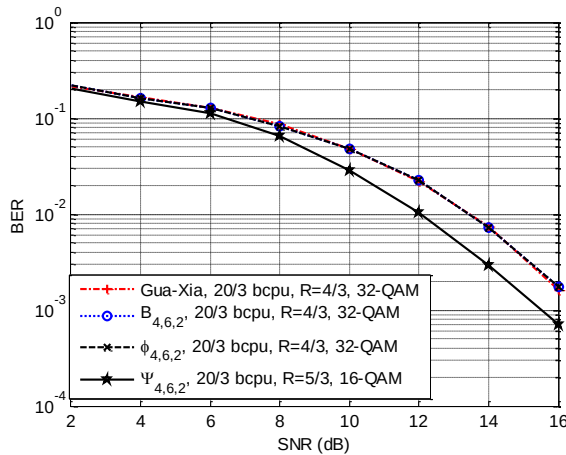


Fig. 1. BER curves of the proposed ($\Psi_{4,6,2}$), Gua-Xia ([13]), $\mathbf{B}_{4,6,2}$ ([16]), and $\Phi_{4,6,2}$ ([17]) codes with four transmit antennas and four receive antennas.

5. Conclusions

In this paper, a new systematic PIC group STBC design was proposed for MIMO system. The proposed code was constructed by embedding conventional PIC group STBC design in the entries of the unbalanced 2-group decodable STBC for any number of transmitted antennas. The new systematic STBC design had higher code rate than the best known PIC group STBC designs. However, the decoding complexity of the transmitted symbols of the proposed code was the same as that of the conventional PIC group STBC design and higher than those of $\mathbf{B}_{M,T,P}$ and $\Phi_{M,T,P}$. In fact, the decoding complexity was

sacrificed and higher code rate was obtained. The simulation results illustrated that the proposed code achieved full diversity and better BER performance than the other PIC group STBC designs at the same bandwidth efficiency. In fact, we sacrificed the decoding complexity and obtain higher code rate. The simulation results illustrated that our proposed code achieves full diversity and better BER performance than the other PIC group STBC designs at the same bandwidth efficiency.

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