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Integration technique method using Kekre transform

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Abstract

In this work, a new approach of the Kekre transform in the integral form is introduced. A theory of operation matrix is developed by numerical computation and by partitioning of the matrices in four submatrices which is proved as the same. An operational matrices of integration based on Kekre transform are applied to the dynamic systems with distributed parameters to analyse their applications. A matrix method is used at the end of this paper which contributes to operation matrix in dynamic systems.

Keywords: *Kekre transform, operational matrix,partitioning matrices, integral technique.*
2010
***Mathematics Subject Classification*:** 42C40, 35C15 , 40C05.

1. Introduction

Transform methods are typically used in many image processing applications such as compression, filtering, enhancement, feature extraction, image texture analysis etc. Using transform domain techniques, it is possible to embed a secret message in different frequency bands of the cover image. Using orthogonal functions to construct the operational matrix for solving and optimization of dynamic systems was first studied by Chen et al. 1977 in [13]. Most commonly used Haar wavelets has been studied by Kekre and Haar as orthogonal functions. In 1997 in [1], the author Chen and Hsiao use the Haar wavelet to explore the new direction in system analysis, establishing an operational matrix for integration via Haar wavelets the drawbacks were eliminated. The authors Chen and Hsaio in 1965 in [6] studied a state-space approach to Walsh series solution of linear systems. Using Fourier series, operational matrix of integration was studied by Paraskevopoulos et al. 1985 in [7]. In transforms applied to Kekre's function in the paper, Laplace Transforms to Kekre's functions 2013 and 2014 in [2,3], Fourier Transforms to Kekre's function, the work can be used for various image processing applications as applied in Sudeep et al. 2008 in [4]. Pathak in his book presented the Wavelet transform in mathematical concepts in 2009 in [12]. The author studied Wavelet transforms versus Fourier transforms by Strang 1993 in [10]. In [9] Haar explained the orthogonal functions and system. The Chang et al. 1984 in [8] and Cheng 1961 in [11], have studied the applications used in the analysis of linear systems. Author Lakshmi Gorty et al. in the study of their work in Laplace Transforms to Kekre's functions 2013 and 2014, Fourier Transforms to Kekre's function, proposed solutions to some problems using Kekre's function and studied Laplace and Fourier transforms using Kekre's function and its applications in form of examples. In continuation of the paper Kekre and Gorty 2013, using inverse Laplace Transforms, the results in the form of Kekre's function were analyzed. In this paper the author presents a new approach of the Kekre transforms in the integral form. Some corollary and theorems are proved in terms of operation matrix in this work. An operational matrix of integration based on Kekre transforms are applied to the distributed parameters dynamic systems to show its applications at the end of this paper.

2. Preliminary Results

Kekre function is defined as

$$K_{a+1}(t) = -(N-a)\left[u\left(t-(a-1)\right)-u\left(t-a\right)\right]+u\left(t-a\right) \quad (2.1)$$

for any order, $a = 1,2,3,...N$ and $a < N; t \geq 0$. Here N is the order of the Kekre's function. First Kekre's function will be always

$$K_1(t) = u(t) \quad (2.2)$$

always for N; with reference to the function Kekre and Gorty in 2013, the generalized Kekre's function is given by

$$K_a(t) = -(N-a+1)\{u(t-(a-2)) - u(t-a+1)\} + u(t-a+1). \quad (2.3)$$

for any order, $a = 2,3,...N$ and $a < N; t \geq 0$. When the order of Kekre's function is 4, the Kekre's function can be represented as:

$$K_1 = u(t)$$
$$K_2 = -3(u(t-0) - u(t-1)) + u(t-1)$$
$$K_3 = -2(u(t-1) - u(t-2)) + u(t-2)$$
$$K_4 = -1(u(t-2) - u(t-3)) + u(t-3)$$

Therefore, they form a transform basis. Any function $y(t)$, which is a square integrable in the interval $(0,1)$, namely $\int_0^1 y^2(t)dt$ is finite. It can be expanded into Kekre transforms as

$$y(t) = c_1k_1(t) + c_2k_2(t) + c_3k_3(t) + c_4k_4(t) + ... = c^tK(t) \quad (2.5)$$

where

$$c_i = 2^j \int_0^1 y(t)k_i(t)\,dt. \quad (2.6)$$

The series expansion as expressed in equation (2.5) contains infinite terms for approximating smooth $y(t)$. If $y(t)$ is piecewise constant by itself, or may be approximated as piecewise constant during each subinterval, then equation (2.5) will be terminated at finite terms. The row vectors are used to denote time functions for input vectors; and coulumn vectors $x, u(t), y(t)$ to

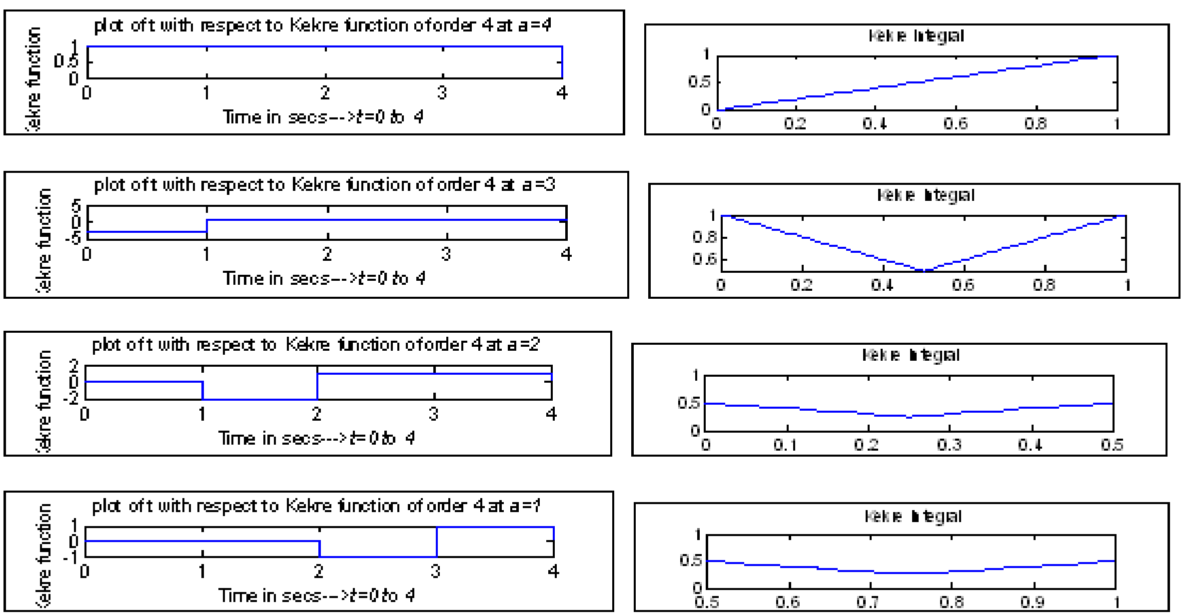

Fig 1. Kekre's function and its corresponding integral

denote the state functions as output vectors.The first four Kekre's function can be expressed as follows:

$$k_1(t) = [1 \quad 1 \quad 1 \quad 1] \quad (2.7)$$

$$k_2(t) = [-3 \quad 1 \quad 1 \quad 1] \quad (2.8)$$

Kekre's transforms are orthogonal to each other as considered by the author in [5]:

$$\int_0^1 k_i(t)k_l(t)\,dt = \delta_{il}. \quad (2.4)$$

$$k_3(t) = \begin{bmatrix} 0 & -2 & 1 & 1 \end{bmatrix} \tag{2.9}$$

$$k_4(t) = \begin{bmatrix} 0 & 0 & -1 & 1 \end{bmatrix} \tag{2.10}$$

$$K_4(t) \triangleq \begin{bmatrix} k_1(t) \\ k_2(t) \\ k_3(t) \\ k_4(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \tag{2.11}$$

The Kekre coefficient c_i can be obtained by applying equation (2.6) directly; also it is more easy to evaluate it by matrix inversion.

$$c^t = y(t) K_4^{-1} \tag{2.12}$$

$$K_4^{-1} = \frac{1}{12} \begin{bmatrix} 3 & -3 & 0 & 0 \\ 3 & 1 & -4 & 0 \\ 3 & 1 & 2 & -6 \\ 3 & 1 & 2 & 6 \end{bmatrix}. \tag{2.13}$$

Equation (2.12) is called the forward transform, which turns the time function $y(t)$ into the coefficient vector c^t and equation (2.5) is known as inverse transform, which recovers $y(t)$ from c^t. Since K_4 and K_4^{-1} contain many zeros, this phenomenon makes the Kekre's Transform much faster than the many renowned transforms.

3. Integration of Kekre transforms

Studying the models of dynamic systems and image processing to understand a continuous system it is required to perform integrations in order to get the dynamic problem solved. The new approach developed here is by using integration technique. Let us consider 4^{th} order system. The integrals of the first four Kekre transforms can be expressed as:

$$\int_0^t k_1(\tau)d\tau = \{t; 0 < t \le 1\} \simeq \frac{1}{8}\begin{bmatrix} 10 & 4 & 2 & 0 \end{bmatrix} \tag{3.1}$$

$$\int_0^t k_2(\tau)d\tau = \begin{Bmatrix} 1-t; 0 < t \le \frac{1}{2} \\ t; \frac{1}{2} < t \le 1 \end{Bmatrix} \simeq \frac{1}{8}\begin{bmatrix} 2 & 0 & 4 & 2 \end{bmatrix} \tag{3.2}$$

$$\int_0^t k_3(\tau)d\tau = \begin{Bmatrix} \frac{1}{2}-t; 0 < t \le \frac{1}{4} \\ t; \frac{1}{4} < t \le \frac{1}{2} \end{Bmatrix} \simeq \frac{1}{8}\begin{bmatrix} 2 & 0 & 0 & 0 \end{bmatrix} \tag{3.3}$$

$$\int_0^t k_4(\tau)d\tau = \begin{Bmatrix} 1-t; \frac{1}{2} < t \le \frac{3}{4} \\ t-\frac{1}{2}; \frac{3}{4} < t \le 1 \end{Bmatrix} \simeq \frac{1}{8}\begin{bmatrix} -1 & 1 & 1 & 1 \end{bmatrix} \tag{3.4}$$

Writing equations from (3.1) to (3.4) , the results obtained are

$$\int_0^t K_4(\tau)d\tau \simeq \frac{1}{8}\begin{bmatrix} 10 & 4 & 2 & 0 \\ 2 & 0 & 4 & 2 \\ 2 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 \end{bmatrix}. \tag{3.5}$$

Let the integrals be expanded into Kekre series. Thus

$$\int_0^t K_4(\tau)d\tau = P_4 K_4(t). \tag{3.6}$$

Here

$$P_4 = \frac{1}{8}\begin{bmatrix} 4 & -2 & -1 & -1 \\ 2 & 0 & 1 & -1 \\ 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}. \tag{3.7}$$

In general for an m^{th} order system with $m = 2^j$, j is a positive integer, the P_m is given as

$$P_m = \frac{1}{2m}\begin{bmatrix} 2mP_{m/2} & -K_{m/2} \\ K_{m/2}^{-1} & 0 \end{bmatrix}. \tag{3.8}$$

To prove matrix equation (3.8) , it is firstly numerically verified. For an m^{th} order system the Kekre matrix K_m is defined with m Kekre functions in row vector form as for matrix equation (3.7) . In other words,

$$K_m(t) = \begin{bmatrix} k_1(t) \\ k_2(t) \\ k_4(t) \\ . \\ . \\ . \\ k_m((t)) \end{bmatrix} \tag{3.9}$$

In other words

$$K_1(t) = [1]; K_1^{-1}(t) = [1] \tag{3.10}$$

$$K_2(t) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}; K_2^{-1}(t) = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}. \tag{3.11}$$

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$$K_4(t)=\begin{bmatrix}1&1&1&1\\-3&1&1&1\\0&-2&1&1\\0&0&-1&1\end{bmatrix};\qquad K_4^{-1}(t)=\frac{1}{12}\begin{bmatrix}3&-3&0&0\\-3&1&-4&0\\3&1&2&-6\\3&1&2&6\end{bmatrix}. \tag{3.12}$$

The operation matrix P_m is the Kekre transform coefficient matrix of these integrals as defined by equation (3.8):

$$P_m=\left[\int_0^t K_m(\tau)d\tau\right]K_m^{-1}(t). \tag{3.13}$$

Thus

$$P_1=\left[\int_0^t K_1(\tau)d\tau\right]K_1^{-1}(t). \tag{3.14}$$

$P_1=\left[\frac{1}{2}\right]$. Thus for $m=2$;

$$P_2=\frac{1}{2}\begin{bmatrix}4P_1&-K_1\\K_1^{-1}&0\end{bmatrix};\ P_2=\frac{1}{4}\begin{bmatrix}2&-1\\1&0\end{bmatrix}. \tag{3.15}$$

Also for $m=4$;

$$P_4=\frac{1}{2.4}\begin{bmatrix}2.4P_2&-K_2\\K_2^{-1}&0\end{bmatrix};$$

$$P_4=\frac{1}{16}\begin{bmatrix}8&-4&-2&-2\\4&0&2&-2\\1&-1&0&0\\1&1&0&0\end{bmatrix}. \tag{3.16}$$

For $m=8$; $P_8=\frac{1}{2.8}\begin{bmatrix}2.8P_4&-K_4(t)\\K_4^{-1}&0\end{bmatrix}$.

$$P_8=\frac{1}{16}\begin{bmatrix}8&-4&-2&-2&-1&-1&-1&-1\\4&0&2&-2&3&-1&-1&-1\\1&-1&0&0&0&2&-1&-1\\1&1&0&0&0&0&1&-1\\\frac{3}{12}&\frac{-3}{12}&0&0&0&0&0&0\\\frac{-3}{12}&\frac{1}{12}&\frac{-4}{12}&0&0&0&0&0\\\frac{3}{12}&\frac{1}{12}&\frac{2}{12}&\frac{-6}{12}&0&0&0&0\\\frac{3}{12}&\frac{1}{12}&\frac{2}{12}&\frac{6}{12}&0&0&0&0\end{bmatrix};$$

$$P_8=\frac{1}{2304}\begin{bmatrix}96&-48&-24&-24&-12&-12&-12&-12\\48&0&24&-24&36&-12&-12&-12\\12&-12&0&0&0&-24&-12&-12\\12&12&0&0&0&0&12&-12\\36&-36&0&0&0&0&0&0\\-36&12&-48&0&0&0&0&0\\36&12&24&-72&0&0&0&0\\36&12&24&72&0&0&0&0\end{bmatrix}. \tag{3.17}$$

All these matrices P_2, P_4, P_8 satisfy equation (3.13).

Theorem and corollary based on operational matrix are given below:

Corollary 3.1 *Let P_a be the transform of the integrals of first two Kekre funtion $k_1(t)$ and $k_2(t)$; Then $P_a=P_{m/2}$* for value of $m=2^j$, consider for $j=2$. (3.18)

Proof. Choose $m=4$; since P_a be the transform of the integrals of first two Kekre funtion $k_1(t)$ and $k_2(t)$, P_4 of equation (3.16) and P_4 of equation (3.7) are equal even though evaluated in two different methods $P_a=\frac{1}{4}\begin{bmatrix}2&-1\\1&0\end{bmatrix}$.

Hence the proof of the corllary.

Corollary 3.2 *Let P_b be the transform of the integrals of Kekre funtion $k_1(t)$ and $k_2(t)$ into the series of $k_3(t)$ and $k_4(t)$. Then*

$$P_b=-\frac{1}{2m}K_{m/2} \tag{3.19}$$

for value of $m=2^j$, consider for $j=2$.

Proof. By definition, P_b is the transform of the integrals of first two Kekre funtion $k_1(t)$ and $k_2(t)$ into the series of $k_3(t)$ and $k_4(t)$.

$$\begin{bmatrix}\int_0^t k_1(\tau)d\tau\\ \int_0^t k_2(\tau)d\tau\end{bmatrix}\simeq\frac{1}{8}\begin{bmatrix}10&4&2&0\\2&0&4&2\end{bmatrix} \tag{3.20}$$

which shows that the Kekre's integral of $k_1(t)$ is a ramp function. Kekre's integral of $k_2(t)$ is a triangular function consisting of rising ramp and falling ramp.

$$P_b = \begin{bmatrix} \int_0^t k_1(\tau)d\tau \\ \int_0^t k_2(\tau)d\tau \end{bmatrix} * \frac{1}{12}\begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 1 & -4 & 0 \\ 3 & 1 & 2 & -6 \\ 3 & 1 & 2 & 6 \end{bmatrix} \quad (3.21)$$

$$= \frac{1}{8}\begin{bmatrix} 10 & 4 & 2 & 0 \\ 2 & 0 & 4 & 2 \end{bmatrix} * \frac{1}{12}\begin{bmatrix} 0 & 0 \\ -4 & 0 \\ 2 & -6 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} -\frac{1}{8} & -\frac{1}{8} \\ \frac{1}{8} & -\frac{1}{8} \end{bmatrix}$$

$$P_b = -\frac{1}{8}K_2 = -\frac{1}{8}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}. \quad (3.22)$$

Hence proved the corollary.

Corollary 3.3 *Let* P_c *be the transform of the integrals of Kekre funtion* $k_3(t)$ *and* $k_4(t)$ *into the series of* $k_1(t)$ *and* $k_2(t)$. *Then*

$$P_c = \frac{1}{2m}K_{m/2}^{-1} \quad (3.23)$$

for value of $m = 2^j$, consider for $j = 2$.

Proof. Here P_c be the transform of the integrals of first two Kekre funtion $k_3(t)$ and $k_4(t)$ into the series of $k_1(t)$ and $k_2(t)$. In numerical expression,the average value is taken to represent these functions:

$$\begin{bmatrix} \int_0^t k_3(\tau)d\tau \\ \int_0^t k_4(\tau)d\tau \end{bmatrix} \simeq \frac{1}{8}\begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 \end{bmatrix}. \quad (3.24)$$

Comparing the first two columns of K_4^{-1} and K_2^{-1} that can be found the former is a dilution of the later. Therefore

$$P_c = \begin{bmatrix} \int_0^t k_3(\tau)d\tau \\ \int_0^t k_4(\tau)d\tau \end{bmatrix} * \frac{1}{12}\begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 1 & -4 & 0 \\ 3 & 1 & 2 & -6 \\ 3 & 1 & 2 & 6 \end{bmatrix}$$

$$\simeq \frac{1}{8}\begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 \end{bmatrix} * \frac{1}{12}\begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 1 & -4 & 0 \\ 3 & 1 & 2 & -6 \\ 3 & 1 & 2 & 6 \end{bmatrix}$$

$$\simeq \frac{1}{8}K_2^{-1}(t) = \frac{1}{16}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

Hence proved. (3.25)

Corollary 3.4 *Let* P_d *be the transform of the integrals of Kekre funtion* $k_3(t)$ *and* $k_4(t)$. *Then* $P_d = 0$ *(null Matrix).*

Proof. Let P_d be the transform of the integrals of Kekre funtion $k_3(t)$ and $k_4(t)$.

$$P_d = \frac{1}{8}\begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 \end{bmatrix} * \frac{1}{12}\begin{bmatrix} 0 & 0 \\ -4 & 0 \\ 2 & -6 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{nullmatrix}. \quad (3.26)$$

It indicates that these rows and coulums are orthogonal to each other. Thus $P_d = 0$ (a null Matrix).

Theorem 3.5 *Let* P_m *be partitioned into four submatrices:* $P_m \triangleq \begin{bmatrix} P_a & P_b \\ P_c & P_d \end{bmatrix}$ (3.27)

where

$$P_a = P_{m/2}$$

$$P_b = -\frac{1}{2m}K_{m/2}$$

$$P_c = \frac{1}{2m}K_{m/2}^{-1}$$

$$P_d = 0 \text{ (nullMatrix)}.$$

Proof. Considering corollary 2.1,2.2,2.3 and 2.4, P_4 can be obtained as

$$P_4 \triangleq \begin{bmatrix} P_a & P_b \\ P_c & P_d \end{bmatrix} \simeq \begin{bmatrix} 1/2 & -1/4 & -1/8 & -1/8 \\ 1/4 & 0 & 1/8 & -1/8 \\ 1/16 & -1/16 & 0 & 0 \\ 1/16 & 1/16 & 0 & 0 \end{bmatrix}.$$

It is same as P_4 calculated by numerical computation in equation (3.7). It completes the proof of equation (3.27).

4. Applications

Example: Consider a unit step voltage is applied to the leakage-free noninductive cable. The voltage $v(x,t)$ and the current $i(x,t)$ must be found. For leakage free non-inductive cable where the conductance is zero and the inductance is zero, equations are written as

$$Ri(x,t) = -\partial v / \partial x \quad (4.1)$$

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$$\frac{\partial^2 v(x,t)}{\partial x^2} = RC\partial v/\partial t \qquad (4.2)$$

with initial conditions $x(0)=0$ and $x'(0)=4$. Equation (4.2) is known as diffussion equation.

Solution: Let Kekre's transforms be used to solve these partial differential equations. Assume $\partial v/\partial t$ can be expanded in a Kekre series as

$$\partial v/\partial t = \mathbf{a}^t(x)K_m(t). \qquad (4.3)$$

Integrating equation (4.3) and applying the integration matrix P_4, from (3.16)

$$v(x,t) = \mathbf{a}^t(x)\int_0^t K_m(t)dt = \mathbf{a}^t(x)P_mK_m(t). \qquad (4.4)$$

Differentiating equation (4.4), the result obtained is

$$\frac{\partial^2 v(x,t)}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left[\mathbf{a}^t(x)P_mK_m(t)\right] = \ddot{\mathbf{a}}^t(x)P_mK_m(t) \qquad (4.5)$$

From equation (4.2) and (4.5) and comparing the values, we obtain

$$RC\partial v/\partial t = \ddot{\mathbf{a}}^t(x)P_mK_m(t) \qquad (4.6)$$

$$RC\mathbf{a}^t(x)K_m(t) = \ddot{\mathbf{a}}^t(x)P_mK_m(t) \qquad (4.7)$$

Thus

$$RC\mathbf{a}^t(x) = \ddot{\mathbf{a}}^t(x)P_m. \qquad (4.8)$$

Since

$$\ddot{\mathbf{a}}^t \simeq \frac{d^2}{dx^2}\mathbf{a}^t(x) \qquad (4.9)$$

The differential equation becomes

$$\left(P_m\frac{d^2}{dx^2} - RC\right)\mathbf{a}^t(x) = 0. \qquad (4.10)$$

Here $\mathbf{a}(x)$ is an m -vector function of x, denotes the space distance. $\mathbf{a}_0$ is constant vector satisfying the boundary conditions.

$$\mathbf{a}^t(x) = \mathbf{a}_0^t(x)e^{-\sqrt{RCP_m^{-1}}x} \qquad (4.11)$$

The voltage $v(x,t)$ is finite as $x \to \infty$.

$$v(x,t) = \mathbf{a}_0^t(x)e^{-\sqrt{RCP_m^{-1}}x}P_mK_m(t). \qquad (4.12)$$

$$Ri(x,t) = -\partial v/\partial x = -\frac{\partial}{\partial x}\mathbf{a}_0^t(x)e^{-\sqrt{RCP_m^{-1}}x}P_mK_m(t). \qquad (4.13)$$

The current $i(x,t)$ is calculated as

$$i(x,t) = -\partial v/\partial x = -\sqrt{\frac{C}{R}}\mathbf{a}_0^t(x)e^{-\sqrt{RCP_m^{-1}}x}\sqrt{P_m}K_m(t). \qquad (4.14)$$

At $x=0$ and $m=1$, the unit step voltage will become

$$v(0,t) = \mathbf{a}_0^t(0)P_1(t)K_1(t) = 1, \qquad (4.15)$$

where

$\mathbf{a}_0^t(0) = [1,0,0,0,....]$. The voltage $v(x,t)$ and the current $i(x,t)$ from equation (4.12) and (4.14) respectively given by

$$v(x,t) = e^{-\sqrt{RCP_m^{-1}}x}. \qquad (4.16)$$

$$i(x,t) = \sqrt{\frac{C}{R}}e^{-\sqrt{RCP_m^{-1}}x}. \qquad (4.17)$$

When $m=1$,

$$P_m^{-1} = P_1^{-1} = 2. \qquad (4.18)$$

When $m=2$,

$$P_m^{-1} = P_2^{-1} = \begin{bmatrix} 0 & 4 \\ -4 & 8 \end{bmatrix}. \qquad (4.19)$$

When $m=4$,

$$P_m^{-1} = P_4^{-1} = \begin{bmatrix} 0 & 0 & 8 & 8 \\ 0 & 0 & -8 & 8 \\ -4 & 4 & 16 & 0 \\ -4 & -4 & 32 & 16 \end{bmatrix}. \qquad (4.20)$$

The corresponding value of voltage $v(x,t)$ and the current $i(x,t)$ is given from (4.16) and (4.17),

For $m=1, v = e^{-\sqrt{RCP_1^{-1}}x}.$

For $m=1, i = \sqrt{\frac{C}{R}}e^{-\sqrt{RCP_1^{-1}}x}.$

The analytic solutions have been described by Cheng in 1961,

$$v(x,t) = erfc\left(\frac{x}{2}\sqrt{\frac{RC}{t}}\right). \qquad (4.21)$$

$$i(x,t) = \sqrt{\frac{C}{Rt}}e^{-x^2\frac{RC}{4t}}. \qquad (4.22)$$

The analytic solution of equation (4.21) and (4.22), can be represented in form of the graph as:

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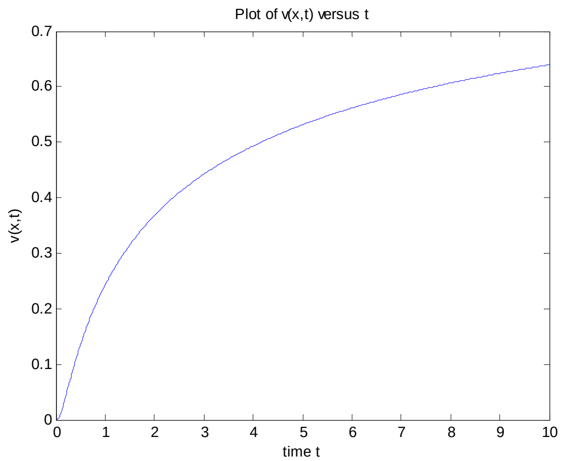

Fig. 2 describing $v(x,t)$ and time t

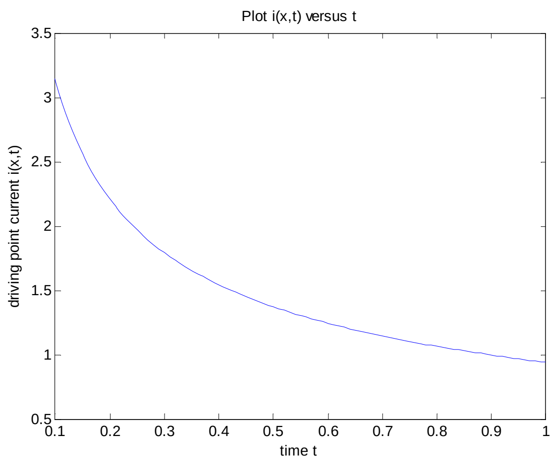

Fig. 3 describes plot of driving current versus time t

Let

$$\mathbf{a}_0^t(x)e^{-\sqrt{RCP_m^{-1}}x} = x_1 \text{ and } \frac{dx_1}{dt} = x_2; \tag{4.23}$$

then equation (4.10) becomes

$$\left(P_m\frac{d^2}{dx^2} - RC\right)x_1 = 0; \tag{4.24}$$

$$\frac{d}{dx}\left(\frac{dx_1}{dt}\right) = RCP_m^{-1}x_1. \tag{4.25}$$

$$\frac{dx_2}{dt} = RCP_m^{-1}x_1. \tag{4.26}$$

Equations (4.25) and (4.26) are written in matrix form

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ RCP_m^{-1} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{4.27}$$

From (4.27) eigenvector for $m=1$ is calculated. Characteristic equation is $\begin{bmatrix} 0 & 1 \\ RC*P_1^{-1} & 0 \end{bmatrix} = 0;$

Let normalised value of RC as 1 sec. $P_1^{-1} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$. The eigenvalues are given by $-\sqrt{2}, \sqrt{2}$. The respective eigenvectors are given as $\left\{\begin{bmatrix} -\frac{1}{2}\sqrt{2} \\ 1 \end{bmatrix}\right\} \leftrightarrow -\sqrt{2}$ and $\left\{\begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 1 \end{bmatrix}\right\} \leftrightarrow \sqrt{2}$. The

Matrix of eigenvectors is given as

$P = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 1 \end{bmatrix}$. $P^{-1} = \begin{bmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2} \end{bmatrix}$. The value of $Pe^{\lambda t}P^{-1}$ can be given as

$$Pe^{\lambda t}P^{-1} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{-\sqrt{2}t} & 0 \\ 0 & e^{\sqrt{2}t} \end{bmatrix}\begin{bmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}e^{\sqrt{2}t} + \frac{1}{2}e^{-\sqrt{2}t} & \frac{1}{4}\sqrt{2}e^{\sqrt{2}t} - \frac{1}{4}\sqrt{2}e^{-\sqrt{2}t} \\ \frac{1}{2}\sqrt{2}e^{\sqrt{2}t} - \frac{1}{2}\sqrt{2}e^{-\sqrt{2}t} & \frac{1}{2}e^{\sqrt{2}t} + \frac{1}{2}e^{-\sqrt{2}t} \end{bmatrix}.$$

By initial conditions $x(0) = 0$ and $x'(0) = 4,$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}e^{\sqrt{2}t} + \frac{1}{2}e^{-\sqrt{2}t} & \frac{1}{4}\sqrt{2}e^{\sqrt{2}t} - \frac{1}{4}\sqrt{2}e^{-\sqrt{2}t} \\ \frac{1}{2}\sqrt{2}e^{\sqrt{2}t} - \frac{1}{2}\sqrt{2}e^{-\sqrt{2}t} & \frac{1}{2}e^{\sqrt{2}t} + \frac{1}{2}e^{-\sqrt{2}t} \end{bmatrix}\begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}e^{\sqrt{2}t} - \sqrt{2}e^{-\sqrt{2}t} \\ 2e^{\sqrt{2}t} + 2e^{-\sqrt{2}t} \end{bmatrix}.$$

$$x_1 = x = \sqrt{2}e^{\sqrt{2}t} - \sqrt{2}e^{-\sqrt{2}t}; x_2 = \frac{dx}{dt} = 2e^{\sqrt{2}t} + 2e^{-\sqrt{2}t}.$$

Solving is x_1 and x_2, the result obtained is given by $x = 2\sqrt{2}\sinh\sqrt{2}t.$

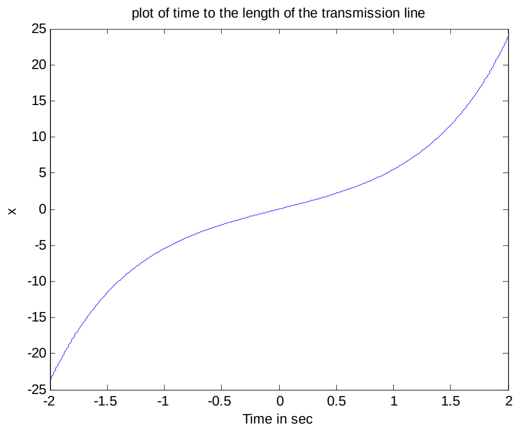

Fig. 4 Plot of t versus x

To find eigenvector for $m = 2.\ P_2^{-1} = \begin{bmatrix} 0 & 4 \\ -4 & 8 \end{bmatrix}.$

Characteristic equation is $\begin{bmatrix} 0 & 1 \\ RC * P_2^{-1} & 0 \end{bmatrix} = 0;$

Let normalised value of RC as 1 sec.

$$P_2^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 4 & 0 \\ -4 & 8 & 0 \end{bmatrix};$$

For eigenvalues: $4, -2i, 2i,$ the eigenvectors are given as

$$\left\{\begin{bmatrix} \frac{1}{4} \\ \frac{5}{8} \\ 1 \end{bmatrix}\right\} \leftrightarrow 4, \left\{\begin{bmatrix} \frac{1}{2}i \\ 0 \\ 1 \end{bmatrix}\right\} \leftrightarrow -2i, \left\{\begin{bmatrix} -\frac{1}{2}i \\ 0 \\ 1 \end{bmatrix}\right\} \leftrightarrow 2i.$$

Matrix of eigenvectors is given as

$$P_2 = \begin{bmatrix} \frac{1}{4} & \frac{1}{2}i & -\frac{1}{2}i \\ \frac{5}{8} & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}; P_2^{-1} = \begin{bmatrix} 0 & \frac{8}{5} & 0 \\ -i & -\frac{4}{5}+\frac{2}{5}i & \frac{1}{2} \\ i & -\frac{4}{5}-\frac{2}{5}i & \frac{1}{2} \end{bmatrix}.$$

The value of $P_2 e^{\lambda t} P_2^{-1}$ is given below as:

$P_2 e^{\lambda t} P_2^{-1}$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{2}i & -\frac{1}{2}i \\ \frac{5}{8} & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} e^{4t} & 0 & 0 \\ 0 & e^{-2it} & 0 \\ 0 & 0 & e^{2it} \end{bmatrix} \begin{bmatrix} 0 & \frac{8}{5} & 0 \\ -i & -\frac{4}{5}+\frac{2}{5}i & \frac{1}{2} \\ i & -\frac{4}{5}-\frac{2}{5}i & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}e^{-2it}+\frac{1}{2}e^{2it} & \frac{2}{5}e^{4t}-\left(\frac{1}{5}+\frac{2}{5}i\right)e^{-2it}-\left(\frac{1}{5}-\frac{2}{5}i\right)e^{2it} & \frac{1}{4}ie^{-2it}-\frac{1}{4}ie^{2it} \\ 0 & e^{4t} & 0 \\ ie^{2it}-ie^{-2it} & \frac{8}{5}e^{4t}-\left(\frac{4}{5}-\frac{2}{5}i\right)e^{-2it}-\left(\frac{4}{5}+\frac{2}{5}i\right)e^{2it} & \frac{1}{2}e^{-2it}+\frac{1}{2}e^{2it} \end{bmatrix}$$

By initial conditions $x(0) = 0$ $x'(0) = 4$ and $x''(0) = 0$

$$\begin{bmatrix} x_1 \\ x_3 \\ x_2 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{2}e^{-2it}+\frac{1}{2}e^{2it} & \frac{2}{5}e^{4t}-\left(\frac{1}{5}+\frac{2}{5}i\right)e^{-2it}-\left(\frac{1}{5}-\frac{2}{5}i\right)e^{2it} & \frac{1}{4}ie^{-2it}-\frac{1}{4}ie^{2it} \\ 0 & e^{4t} & 0 \\ ie^{2it}-ie^{-2it} & \frac{8}{5}e^{4t}-\left(\frac{4}{5}-\frac{2}{5}i\right)e^{-2it}-\left(\frac{4}{5}+\frac{2}{5}i\right)e^{2it} & \frac{1}{2}e^{-2it}+\frac{1}{2}e^{2it} \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} ie^{-2it}-ie^{2it} \\ 0 \\ 2e^{-2it}+2e^{2it} \end{bmatrix}.$$

Thus

$$x_1 = x = ie^{-2it} - ie^{2it} = 2\sin 2t$$

and

$$x_2 = \frac{dx}{dt} = 2e^{-2it} + 2e^{2it} = 4\cos 2t.$$

5. Conclusion

A new approach of the Kekre transform in the integral form is introduced in this paper. A theory of operation matrix is developed by numerical computation and by partitioning of the matrices in four submatrices which is proved as the same. An operational matrices of integration based on Kekre transform have been analysed with their applications. A matrix method is used at the end of this paper which contributes to operation matrix in dynamic systems.

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