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# A New Heuristic Algorithm for Constructing Steiner Trees inside Simple polygons in Presence of Obstacles

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**Abstract**

Steiner tree problem leads to solutions in several scientific and business contexts, including computer networks routing and electronic integrated circuits. Computing fields of this problem has become an important research topic in computational geometry. Considering the number of points in the Euclidean plane, called terminal points, a minimum spanning tree is obtained which connects these points. A series of other points (Steiner points) are added to the tree, which makes it shorter in length. The resulting tree is called Euclidean Steiner minimal tree. It is considered as an NP-hard problem. Considering a simple polygon P with m vertices and n terminals, in which you are trying to find a Euclidean Steiner tree that is connected to all n terminals existing inside p. In this paper we propose a solution for several terminals in a simple polygonal in presence of obstacles.

***Keywords***: *Euclidean Steiner Minimal Tree, Straight skeleton of simple polygon, geodesic convex hull.*

## 1. Introduction

Steiner problem can be used in scientific and commercial fields such as computer networks routing and integrated electronic circuits, oil distribution and transport network. Computational fields of this problem make it an important research topic in computational geometry. Considering a few points in Euclidean plane, the shortest Path connecting these points lead to the attainment of a tree is called Euclidean Steiner minimal tree. Euclidean Steiner minimal tree is considered as an NP-hard problem [1]. In this paper, we obtain a Euclidean Steiner minimal tree (ESMT), which has n terminals in a simple polygon with m vertices and several obstacles. It has been proven that ESMTs connecting 4 terminals together have high-quality [4] Lee et al.solutions for cases with no obstacles [2,3]. proposed an a algorithm for the ESMT within a simple polygon with O(K log k), where K = m + n. They subset of each of these terminals insidedetermined 4, 3, 2 a simple polygon. Steiner tree is obtained from any have provided an exact subsets' vertices. Winter et al. algorithm for the three terminals in a simple polygon in 4 terminal in a time O(K) [5], an exact algorithm for heuristic algorithm for over 4 terminalsimple polygon [6], [7]. The proposed algorithm in this paper can solve the Steiner tree in a simple polygon with some obstacles. Finally, we compare our results with the data and results presented in [13]. This paper is organized as follows: Section 2 is dedicated to some basic definitions. The proposed algorithm is introduced in section 3. The computational results are presented in section 4. The final section brings other sections in a conclusion.

## 2. Basic definitions

A polygon $P$ is simple if it is not self-intersecting and its interior $i(p)$ is not empty and connected. A point $P$ is said to be in $P$ if $p \in i(p) \cup p$ .A vertex $v$ on $P$ is convex if its interior angle is less than 180°.

Otherwise, it is reflex. A reflex vertex is said to be wide if its interior angle is at least 240°. Clockwise successor and predecessor vertices of a vertex $v$ are denoted by $v^+$ and $v^-$, respectively. In order to simplify some proofs, it is assumed that $v^-v$ and $vv^+$ are not colinear for any $v \in p$ . A simple polygon is called a $c-$kite iff precisely $c$ of its vertices is convex. Boundaries of a $c-$kite $P$ between two consecutive convex vertices are referred to as sides of $P$ . A polygon $P$ is weakly-simple if it is not self-intersecting. In particular, a weakly-simple polygon can have empty or disconnected interior.

The shortest path between two points $u$ and $v$ in a polygon $P$ will be denoted by $P(u,v)$. $P(u,v)$ is a unique polygonal chain and its interior vertices are reflex vertices of $P$. A line $L$ is said to be an interior tangent of a c-kite $P$ at a touch vertex $v \in P$ iff one of the following cases occurs.

- $v$ is a reflex vertex, and edges $v^-v$ and $vv^+$ are on the same side of $L$.
- $v$ is a convex vertex, and edges $v^-v$ and $vv^+$ are on the opposite sides of $L$.
- $v^-v$ overlaps with $L$ .

An interior tangent $L$ with a touch-point $v$ is oriented in such a way that the edge $vv^+$ is on its left. Two interior tangents of a c-kite $P$ are distinct if they have different slopes or different touch vertices. Consider a reflex vertex v of a c-kite P . let $q_v^-$ and $q_v^+$ denote the convex vertex that is reached form v by moving counterclockwise and (respectively clockwise) on P. Let sv denote an edge in p overlapping with an interior tangent of v . Only one of the vertices $v^-$ and $v^+$ is visible form s. Let $q_v^s$ denote the convex vertex that can be reached form v by moving counterclockwise on P in $v^-$ is invisible from s, and by moving clockwise if $v^+$ is invisible from s. if v is convex, let $q_v^s = v$ .

An ESMT inside a simple polygon cannot have vertices of degree greater than three. Vertices of degree 3 are called Steiner points if they are located in the interior of P . The edges incident to Steiner points make 120° with each other. They are called degenerate Steiner points if they are located on the boundary of P. Degenerate Steiner points can only occur at wide reflex vertices of P. The reader is referred to [8] for basic definitions and properties of ESMTs.

## 2.1. Polygon Reductions

Consider a unique polygon $P'$ inside P containing the terminals Z, and such that its perimeter is as short as possible. Provan [9] proved that there always exists an ESMT for Z in P completely in$P'$. Toussaint [10] gave an $O(n(\log n + \log k) + k)$ algorithm to determine $P'$. The complexity of this algorithm reduces to $O(k)$ if n is fixed. $P'$ is sometimes referred to as the geodesic convex hull for its polygon and its terminals.

## 2.2. Straight skeleton

In geometry, a straight skeleton is a method of representing a polygon by a topological skeleton. It is similar in some ways to the medial axis but differs in that the skeleton is composed of straight line segments, while the medial axis of a polygon may involve parabolic curves. Straight skeletons were first defined for simple polygons by Aichholzer et al. and generalized to planar straight line graphs by Aichholzer and Aurenhammer [11, 12]. The straight skeleton of a polygon is defined by a continuous shrinking process in which the edges of the polygon are moved inwards parallel to themselves at a constant speed. As the edges move in this way, the vertices where pairs of edges meet also move, at speeds that depend on the angle of the vertex. If one of these moving vertices collides with a nonadjacent edge, the polygon is split in two by the collision, and the process continues in each part. The straight skeleton is the set of curves traced out by the moving vertices in this process [11, 12]. Figure 1 of the illustration shows the straight skeleton of a polygon.

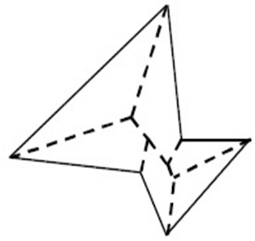

Figure 1. Straight skeleton of simple polygon

## 3. The Proposed Algorithm Steps

A simple polygon with m vertices, n terminals and a of obstacle is presented as an input, as shown in Figure 2.

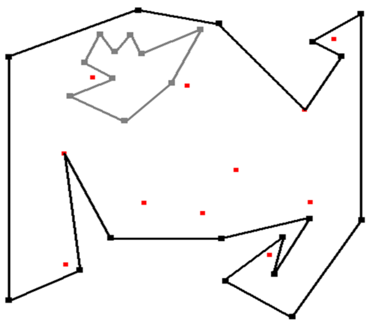

Figure 2. A simple polygon m vertices, n terminals and a of obstacle

Our proposed algorithm has three steps.

**Step1**. First the geodesic convex hull of the terminals in a simple polygon is obtained, which is shown in Figure 3.

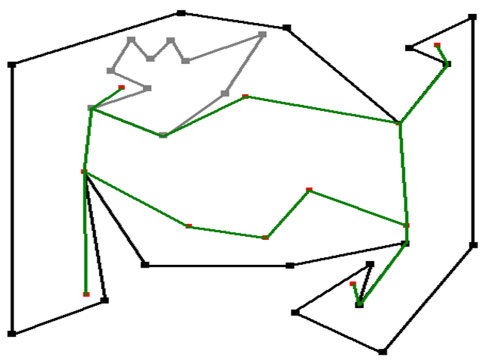

Figure 3. Geodesic convex hull of the terminals in a simple polygon

When dealing with obstacles, the following procedure is conducted:
Assume that obtaining the shortest distance between two terminals of A and B, where there is an obstacle between them is intended. First, the points of tangency of A and B with the obstacle are obtained, then there are two paths for connecting A to B from the points of tangency, which the shorter path is selected, as shown in Figure 4.

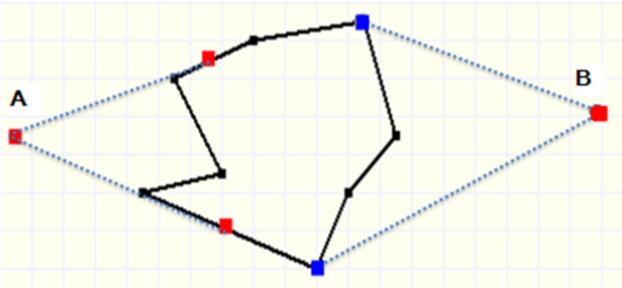


Figure 4. An obstacle between the two terminals of A and B

**Step2**. Draw straight skeleton for the geodesic convex hull and find candidates of Steiner points. It is shown in Figure 5.

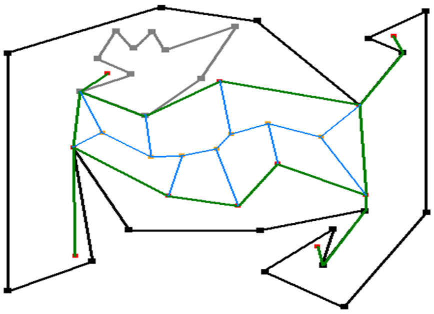

Figure 5. The straight skeleton of geodesic convex hull

**Step3.** A tree is obtained with the geodesic convex hull skeleton which contains candidate Steiner points. Non-terminal leaves and connected edges are eliminated in this tree which may convert it into several separated parts. The shortest distance between two Steiner points is obtained using the Kruskal's algorithm and then the Steiner points are connected to each other. The resulting tree is Euclidean Steiner minimal tree with obstacle, which is shown in Figure 6.

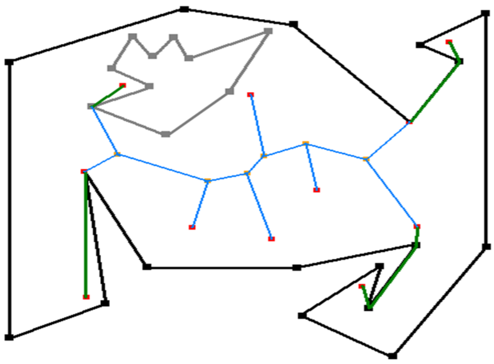

Figure 6. Euclidean Steiner minimal tree with obstacle

Our algorithm can also handle the geodesic convex hull with Obstacles as long as they have the appropriate orientation. That means the geodesic convex hull interior lays leftwards from all of the line segments, vertices of the outer boundary are in counter-clockwise order and the vertices of the holes are in clockwise order. An example of the straight skeleton of a geodesic convex hull with a Obstacle is in Figure 7.

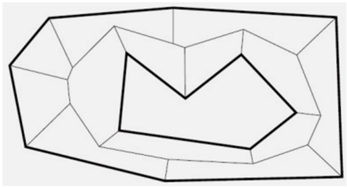

Figure 7. An example of the straight skeleton of a geodesic convex hull with a Obstacle

## 4. Computational Results

We implemented the proposed algorithm in Delphi programming language and performed our experiments with examples of Soukup [13]. We considered a convex polygon around all the terminals and assumed that there is no terminal in the obstacles. Then we compared some of our results with optimum results in table 1. The presented algorithm provides good results as shown in Table 1.

TABLE 1: PROPOSED ALGORITHM COMPARED WITH SOUKUP EXAMPLES

| **Example number** | ***Optimal result*** | ***Our proposed algorithm*** |
|---|---|---|
| EX.1 | 166.44 | 166.44 |
| EX.2E | 220.53 | 220.53 |
| EX.2F | 217.78 | 219.90 |
| EX.3 | 159.88 | 163.80 |
| EX.4 | 127.41 | 130.96 |
| EX.5 | 164.83 | 168.21 |
| EX.6 | 127.34 | 128.71 |

## 5. Conclusion

This paper presents an algorithm capable of solving Steiner tree problem inside a simple polygon with some obstacles in the Euclidean plane. Computational results of stated algorithm presented. Resulting algorithm is simple in terms of implementation and leads to good results.

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